

# THE MIDPOINT FORMULA

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## THE MIDPOINT FORMULA

The midpoint of the line segment between points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by the Midpoint Formula:

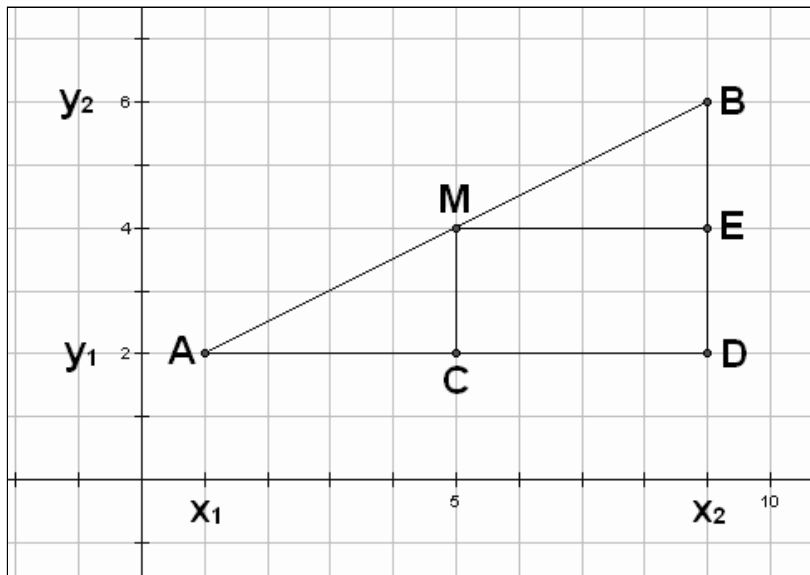
$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Here,  $x_1$  (read as ‘ $x$  sub 1’) denotes the  $x$ -value of the first point, and  $y_1$  (read as ‘ $y$  sub 1’) denotes the  $y$ -value of the first point. Similarly,  $x_2$  and  $y_2$  denote the  $x$ -value and  $y$ -value of the second point.

Thus, to find the location that is exactly halfway between two points, you average the  $x$ -values, and average the  $y$ -values.

The Midpoint Formula follows easily from the following observations:

- The average of two numbers always lies exactly halfway between the two numbers.
- Referring to the sketch below,  $\triangle ABD$  is similar to  $\triangle AMC$ .  
That is, these two triangles have the same angles.  
Why? They both share angle  $A$ , and they both have a right angle.  
Since all the angles in a triangle sum to  $180^\circ$ , the third angles must also be the same.
- Similarity gives us what we need!  
It tells us that  $\triangle ABD$  and  $\triangle AMC$  have exactly the same shapes—they're just different sizes.  
Since  $\overline{AM}$  is exactly half of  $\overline{AB}$ ,  $\overline{AC}$  must be exactly half of  $\overline{AD}$ .  
Thus,  $C$  is the midpoint between  $A$  and  $D$  (which can be found by averaging  $x_1$  and  $x_2$ ).
- Use a similar argument to show that  $\overline{DE}$  (which has the same length as  $\overline{CM}$ ) is exactly half of  $\overline{DB}$ .



**EXAMPLES:**

**Question:**

Find the midpoint of the line segment between  $(1, -3)$  and  $(-2, 5)$ .

**Solution:**

$$\left( \frac{1 + (-2)}{2}, \frac{-3 + 5}{2} \right) = \left( -\frac{1}{2}, 1 \right)$$

**Question:**

Suppose that  $(2, 3)$  is exactly halfway between  $(-1, 5)$  and  $(x, y)$ .

Find  $x$  and  $y$ .

**Solution:**

Rephrasing,  $(2, 3)$  is the midpoint of the segment with endpoints  $(-1, 5)$  and  $(x, y)$ .

Thus:

$$(2, 3) = \left( \frac{-1 + x}{2}, \frac{5 + y}{2} \right) \quad \text{use the Midpoint Formula}$$

$$2 = \frac{-1 + x}{2} \quad \text{and} \quad 3 = \frac{5 + y}{2} \quad \begin{array}{l} \text{for ordered pairs to be equal,} \\ \text{the first coordinates must be equal} \\ \text{and the second coordinates must be equal} \end{array}$$

$$4 = -1 + x \quad \text{and} \quad 6 = 5 + y \quad \begin{array}{l} \text{clear fractions} \\ \text{(multiply both sides of both equations by 2)} \end{array}$$

$$5 = x \quad \text{and} \quad 1 = y \quad \text{finish solving each equation}$$

$$x = 5 \quad \text{and} \quad y = 1 \quad \text{write your solutions in the conventional way}$$